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BRST cohomology of the Chapline-Manton model

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Abstract

We completely compute the local BRST cohomology $H(s|d)$ of the combined Yang-Mills-2-form system coupled through the Yang-Mills Chern-Simons term ("Chapline-Manton model"). We consider the case of a simple gauge group and explicitely include in the analysis the sources for the BRST variations of the fields ("antifields"). We show that there is an antifield independent representative in each cohomological class of $H(s|d)$ at ghost number 0 or 1. Accordingly, any counterterm may be assumed to preserve the gauge symmetries. Similarly, there is no new candidate anomaly beside those already considered in the literature, even when one takes the antifields into account. We then characterize explicitly all the non-trivial solutions of the Wess-Zumino consistency conditions. In particular, we provide a cohomological interpretation of the Green-Schwarz anomaly cancellation mechanism.

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1 Introduction

Chern-Simons couplings of two-forms fields to Yang-Mills gauge fields play a central role in the Green-Schwarz anomaly cancellation mechanism [1] and, for this reason, are important in string theory [2]. In this letter we completely work out the general solution of the Wess-Zumino consistency condition at all ghost numbers (BRST cohomology $H(s|d)$) in the space of local exterior n-forms depending on the fields and the antifields, for the Chapline-Manton model whose Lagrangian reads [3, 4, 5, 6],

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} - \frac{1}{12}H_{\mu\nu\rho}H^{\mu\nu\rho}, \qquad (1.1)\\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - C^a_{bc}A^b_\mu A^c_\nu, \qquad (1.2)\\ H_{\mu\nu\rho} &= \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu} - 2\lambda C_{abc}A^a_\mu A^b_\nu A^c_\rho \\ &\quad -2\lambda(A^a_\mu F_{a\nu\rho} + A^a_\nu F_{a\rho\mu} + A^a_\rho F_{a\mu\nu}). \qquad (1.3) \end{aligned}$$

Here A^a_μ is the Yang-Mills vector potential, $B_{\mu\nu}$ is an abelian two-form and λ is the coupling constant. The $H_{\mu\nu\rho}$ are the components of the exterior form $H = dB + \lambda\omega_3$ where $\omega_3 = tr(AF + \frac{1}{3}A^3)$ is the Chern-Simons three-form. For definiteness, the gauge group G is taken to be simple.

Our main result is that the antifields can be completely eliminated in cohomology at ghost number zero and one. This implies that:

- there are no new antifield-dependent anomalies and the only possible anomalies of the coupled system are those of the pure Yang-Mills theory that are not made trivial by the coupling to the two-form;

- the counterterms may always be chosen so as to preserve the gauge symmetries. Thus, structural constraints of the type considered in [7] can be consistently imposed.

In order to establish this result, we follow the approach developed in [8] for the pure Yang-Mills theory. This is made possible by a change of variables that brings the BRST differential s to the same form $s = \delta + \gamma$ (without extra higher order contributions). Here, δ is the Koszul-Tate differential associated with the gauge covariant equations of motion while γ is, up to inessential terms, the coboundary operator of the Lie algebra cohomology of G in some definite representation space (sections 2 and 3).

Once s has been brought to the form $s = \delta + \gamma$, the computation of the cohomology $H(s|d)$ of s modulo the spacetime exterior derivative d proceeds by expanding the cocycles according to the antighost number. The obstructions for removing the antifields from a given cocycle lie in the groups $H(\delta|d)$ of the "characteristic cohomology", as in the pure Yang-Mills case. We thus compute $H(\delta|d)$, which has been related in [9] to the conservation laws of first and higher orders. We find that the conservation laws for the Chapline-Manton model are of only two types: conserved tensor $H^{\lambda\mu\nu}$ of rank three, $\partial_\lambda H^{\lambda\mu\nu} \approx 0$, and ordinary conservation laws of rank one associated with rigid symmetries (e.g., Poincaré symmetries). The corresponding obstructions cannot arise at ghost number zero or one, for which one can accordingly always remove the antifields by adding exact terms (section 4).

The other important ingredient of the analysis is that all the solutions of the Wess-Zumino consistency conditions involving $trC^3 \equiv C_{abc}C^aC^bC^c$ (where C^a is the Yang-Mills ghost) and trF^2, or related to them through the descent equations, are removed from the BRST cohomology. This is because trC^3 is s-exact when the coupling is turned on, $trC^3 = (3/\lambda)s\rho$, where ρ is the ghost of ghost associated to the reducibility of the 2-form gauge symmetries. In the same way, trF^2 is d-exact in the space of gauge invariant exterior forms, $trF^2 = (1/\lambda)dH$. These properties are at the core of the Green-Schwarz anomaly cancellation mechanism and are discussed in section 5.

2 BRST differential

According to the standard rules of the BRST formalism [10, 11, 12] we introduce, beside the fields $(A^a_\mu, B_{\mu\nu})$, the antifields $(A^{*\mu}_a, C^*_a, B^{*\mu\nu}, \eta^{*\mu}, \rho^*)$ and the ghosts (C^a, η_μ, ρ), with Grassmann parity given by,

$$\epsilon(A^{*\mu}_a) = \epsilon(B^{*\mu\nu}) = \epsilon(\rho^*) = 1;\ \epsilon(C^*_a) = \epsilon(\eta^{*\mu}) = 0. \tag{2.1}$$

The action of the BRST differential s on the algebra $\mathcal{P}$ of spacetime forms with coefficients that are polynomials in the fields, antifields, ghosts and their derivatives is defined through [11, 12]

$$sA^a_\mu = \partial_\mu C^a - C^a_{bc}A^b_\mu C^c = D_\mu C^a, \tag{2.2}$$

$$sC^a = \frac{1}{2}C^a_{bc}C^bC^c, \tag{2.3}$$

$$
\begin{aligned}
sB_{\mu\nu} &= 2\lambda C_a(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + \partial_\mu \eta_\nu - \partial_\nu \eta_\mu, & (2.4)\\
s\eta_\mu &= \lambda C_{abc} C^a C^b A^c_\mu - \partial_\mu \rho, & (2.5)\\
s\rho &= \frac{1}{3}\lambda C_{abc} C^a C^b C^c, & (2.6)\\
sA^{*\mu}_a &= D_\nu F^{\nu\mu}_a + 2\lambda H^{\mu\nu\rho} F_{a\nu\rho} - 2\lambda \partial_\rho H^{\rho\mu\nu} A_{a\nu} & \\
&\quad -2\lambda \partial_\mu (B^{*\nu\mu} C_a) - \lambda \eta^{*\mu} C_{abc} C^b C^c + C_{abc} A^{*b\mu} C^c, & (2.7)\\
sC^*_a &= 2\lambda B^{*\mu\nu} \partial_\mu A_{a\nu} + 2\lambda C_{abc} \eta^{*\mu} C^b A^c_\mu + \lambda C_{abc} \rho^* C^b C^c & \\
&\quad -D_\mu A^{*\mu}_a - C_{abc} C^{*b} C^c, & (2.8)\\
sB^{*\mu\nu} &= \partial_\rho H^{\rho\mu\nu}, & (2.9)\\
s\eta^{*\mu} &= -\partial_\nu B^{\nu\mu}, & (2.10)\\
s\rho^* &= -\partial_\mu \eta^{*\mu}. & (2.11)
\end{aligned}
$$

The action of s on the fields and the ghosts takes a clearer form when rewritten in differential form notations,

$$sA + DC = 0, \tag{2.12}$$
$$sC = C^2, \tag{2.13}$$
$$sB + \lambda\omega_2 + d\eta = 0, \tag{2.14}$$
$$s\eta + \lambda\omega_1 + d\rho = 0, \tag{2.15}$$
$$s\rho = \frac{1}{3}\lambda tr C^3. \tag{2.16}$$

Here, the one-form ω_1 and the two-form ω_2 are related to the Chern-Simons form ω_3 through the descent,

$$
\begin{aligned}
s\omega_3 + d\omega_2 &= 0, \quad \omega_2 = tr(CdA), & (2.17)\\
s\omega_2 + d\omega_1 &= 0, \quad \omega_1 = tr(C^2 A), & (2.18)\\
s\omega_1 + d(\frac{1}{3} tr C^3) &= 0. & (2.19)
\end{aligned}
$$

In terms of the above variables, the BRST differential has two major defects. The first is that it has a component of antighost number 1. There are indeed terms of "higher order" in s [12], e.g. $\eta^{*\mu} C_{abc} C^a C^b$ in (2.7). Consequently, the BRST differential does not split as the sum of the Kozsul-Tate differential and the longitudinal exterior derivative as it does when the fields are not coupled. The second undesired feature is that the BRST

variations of the antifields of the Yang-Mills sector contain contributions not covariant under the gauge transformations, e.g. $\partial_\rho H^{\rho\mu\nu} A_{a\nu}$ in (2.7). One can remedy both problems by redefining the antifields of the Yang-Mills sector according to the following invertible transformations:

$$A_a^{*\mu} \rightarrow A_a^{*\mu} + 2B^{*\mu\nu} A_{a\nu} - 2\eta^{*\mu} C_a, \tag{2.20}$$

$$C_a^* \rightarrow C_a^* + 2\eta^{*\mu} A_{a\mu} - 2\rho^* C_a. \tag{2.21}$$

In terms of the new variables, the BRST differential takes the familiar form,

$$s = \delta + \gamma, \tag{2.22}$$

with:

$$\delta B^{*\mu\nu} = \partial_\rho H^{\rho\mu\nu}; \quad \delta \eta^{*\mu} = -\partial_\nu B^{*\nu\mu}; \quad \delta\rho^* = -\partial_\mu \eta^{*\mu}; \tag{2.23}$$

$$\delta A_a^{*\mu} = D_\nu F_a^{\nu\mu} + 2\lambda H^{\mu\nu\rho} F_{a\nu\rho}; \quad \delta C_a^* = 2\lambda B^{*\mu\nu} F_{a\mu\nu} - D_\mu A_a^{*\mu}, \tag{2.24}$$

and

$$\gamma B^{*\mu\nu} = \gamma\eta^{*\mu} = \gamma\rho^* = 0; \quad \gamma A_a^{*\mu} = C_{abc} A^{*b\mu} C^c; \quad \gamma C_a^* = -C_{abc} C^{*b} C^c; \tag{2.25}$$

$$\gamma\ (fields) = s\ (fields). \tag{2.26}$$

The γ variations of the Yang-Mills variables are now identical to those of the uncoupled theory and show that A_μ^{*a} and C_a^* transform according to the adjoint representation.

3 Cohomology of δ and γ

3.1 $H(\delta)$

The new antifields defined by (2.20) and (2.21) are no longer homogenous in the standard antighost number, since they mix antifields with different antighost numbers. We thus redefine the antighost number as

$$antigh(fields) = antigh(ghosts) = 0; \tag{3.1}$$

$$antigh(A_a^{*\mu}) = 1; \quad antigh(C_a^*) = 2; \tag{3.2}$$

$$antigh(B^{*\mu\nu}) = 1; \quad antigh(\eta^{*\mu}) = 2; \quad antigh(\rho^*) = 3. \tag{3.3}$$

In terms of the new antighost number, the differential δ defined by (2.23) - (2.25) has antighost -1 and the differential γ has antighost number zero. From now on, the antighost number will always refer to (3.1) - (3.3).

The initial equations of motion are not manifestly covariant under the internal gauge symmetries since they contain a "bare" A. The redefinition (2.20) - (2.21) replaces these equations by linear combinations of them which are, by contrast, manifestly covariant. The new equations are clearly equivalent to the original ones and are obtained by equating to zero the δ-variations of the new antifields of antighost number 1,

$$\partial_\rho H^{\rho\mu\nu} = 0, \quad D_\nu F_a^{\nu\mu} + 2\lambda H^{\mu\nu\rho} F_{a\nu\rho} = 0. \tag{3.4}$$

Similarly, the reducibility identities on the equations of motion encoded in the Koszul-Tate variations of the antifields of antighost number 2 are also manifestly invariant under gauge transformations. For this reason, one can call δ the "covariant Koszul-Tate differential". Because δ encodes a complete set of equations of motion and reducibility identitites, one has the standard result,

Theorem 3.1 *$H_i(\delta) = 0$ for $i > 0$, where i is the antighost number, i.e, the cohomology of δ vanishes in antighost number strictly greater than zero.*

In degree zero, the cohomology of δ is the algebra of "on-shell functions" [13, 14, 12, 15].

3.2 $H(\gamma)$

One also associates to γ another grading called the 'pureghost number', which is given by,

$$puregh(fields) = puregh(antifields) = 0; \tag{3.5}$$

$$puregh(C^a) = 1; \quad puregh(\eta_\mu) = 1; \quad puregh(\rho) = 2. \tag{3.6}$$

The 'ghost' number is then defined as the difference between the pureghost number and the antighost number, $gh = puregh - antigh$.

When $\lambda = 0$ (uncoupled case), the cohomology of γ, $H(\gamma)$, is given by the tensor product of the pure Yang-Mills cohomology and of the free 2-form cohomology which have already been calculated separately in [16, 17, 18]. Their

results can be stated as follows. Let the variables χ_0 denote collectively (i) the Yang-Mills field strengths, their covariant derivatives $D_{\alpha_1} \dots D_{\alpha_k} F^a_{\mu\nu}$, the antifields and their covariant derivatives $D_{\alpha_1} \dots D_{\alpha_k} A^{*\mu}_a$,$D_{\alpha_1} \dots D_{\alpha_k} C^*_a$; these transform according to the adjoint representation; and (ii) the free 2-form field strengths $H^0_{\mu\nu\rho} = (dB)_{\mu\nu\rho}$, their derivatives, the antifields $B^{*\mu\nu}$, $\eta^{*\mu}$, ρ^*, their derivatives and the undifferentiated ghost of ghost ρ. Then the representatives of $H(\gamma)$ can be written as $a = \sum_J \alpha_J(\chi_0)\omega^J(C^a)$, where the $\alpha_J(\chi_0)$ are invariant polynomials in the χ_0 and where the $\omega^J(C^a)$ constitute a basis of the Lie algebra cohomology of the Lie algebra of the gauge group. The ω^J are polynomials in the so-called "primitive forms", i.e trC^3, trC^5 if $trC^5 \neq 0$, etc. [For instance, for $SU(3)$, the $\omega^J(C^a)$ can be taken to be $\{1$, trC^3, trC^5, $trC^3\ trC^5\}$].

When the Chern-Simons coupling is turned on ($\lambda \neq 0$), the results are very similar but there are however two modifications: (i) one must replace in the above cocycles the free field strengths $H^0_{\mu\nu\rho}$ and their derivatives by the modified invariant field strengths $H_{\mu\nu\rho}$ (1.3) and their derivatives (we shall denote the new set of improved variables defined in this manner by χ); (ii) the ghost of ghost ρ and the primitive form trC^3 drop from the cohomology since these elements now obey the relation $\gamma\rho = \frac{\lambda}{3} trC^3$, which indicates that trC^3 is exact, while ρ is no longer closed. This last feature underlies the Green-Schwarz anomaly cancellation mechanism. We thus have:

Theorem 3.2 *The representatives of $H(\gamma)$ can be written,*

$$a = \sum_J \alpha_J(\chi)\overline{\omega}^J(C^a), \tag{3.7}$$

where the χ stand for the field strengths ($F^a_{\mu\nu}$ $H_{\mu\nu\rho}$), the antifields ($A^{*\mu}_a$, C^*_a, $B^{*\mu\nu}$, $\eta^{*\mu}$, ρ^*) and their covariant derivatives, and $\overline{\omega}^J(C^a)$ is the subset of the $\omega^J(C^a)$ which does not depend on trC^3.

[So, for $SU(3)$, the $\overline{\omega}^J(C^a)$ are just $\{1, trC^5\}$].

4 Antifield Dependence

The local cohomology $H^n_g(s|d)$ in maximal form degree n - the only one considered here - and ghost number g is obtained by solving the Wess-Zumino consistency condition,

$$sa_g + db_{g+1} = 0, \tag{4.1}$$

where a_g and b_{g+1} are respectively local n- and $(n-1)$-forms. One must further identify solutions which differ by s-exact and d-exact terms (trivial terms), i.e, $a_g \sim a'_g = a_g + sn_{g-1} + dm_g$.

Possible anomalies are elements of $H(s|d)$ for $gh = 1$ and counterterms correspond to $gh = 0$. Furthermore, if a counterterm is independent of the antifields, then the gauge symmetries are preserved when this counterterm ("deformation") is added to the action [19, 7].

Our strategy for investigating the Wess-Zumino consistency condition is identical to the one used in [8] for the pure Yang-Mills case. Any solution a can be decomposed according to the antighost number, $a = a_0 + \ldots + a_k$, $antigh(a_k) = k$. In antighost $k+1$, Equation (4.1) reads: $\gamma a_k + db_k = 0$. Just as in [8], it is easy to see that by an allowed redefinition of a one can choose a_k such that $\gamma a_k = 0$ and $b_k = 0$ if $k > 0$. By theorem **3.2** we thus have $a_k = \sum_J \alpha_J(\chi)\overline{\omega}^J(C^a)$. Next one shows that $\alpha_J(\chi)$ has to obey the equation $\delta\alpha_J + d\beta_J = 0$ and so defines a cycle of the invariant characteristic cohomology. Were α_J a trivial solution, i.e, $\alpha_J = \delta\mu_J + d\nu_J$, then a_k could be eliminated by an allowed redefinition of a. The obstructions to the removal of the antifields are therefore elements of the (invariant) characteristic cohomology, which describes the conservation laws of the theory [9].

4.1 Invariant characteristic cohomology

The ordinary characteristic cohomology $H(\delta|d)$ and the invariant characteristic cohomology have already been separately studied in detail in antighost number > 1 for the Yang-Mills case and for the p-form case [9, 8, 18]. For the Yang-Mills case, both cohomologies have been shown to vanish; for a 2-form, they are given by ρ^*, which has antighost 3. The corresponding conservation law reads $\partial_\mu \partial^{[\mu} B^{\nu\rho]} \approx 0$. For the coupled system, one can verify that the ordinary cohomology for $k > 1$ is still given by ρ^*, i.e., the coupling does not introduce any new cohomology and does not remove ρ^* [1]. The result holds also for the invariant cohomology:

Theorem 4.1 *In antighost $k > 1$, any invariant solution $\alpha_J(\chi)$ of the equa-*

[1]This follows from the results of [9], sections 10 and 11. Note that the isomorphism $H_k(\delta|d) \simeq H^{-k}(s|d)$ $(k \geq 1)$ determines the local BRST cohomolgy at negative ghost number.

tion $\delta\alpha_J + d\beta_J = 0$ can be written,

$$\alpha_J = k_J \rho^* + \delta\mu_J + d\nu_J, \tag{4.2}$$

where μ_J and ν_J are invariant and where k_J are constants.

Proof: The proof is based on a "spectral sequence argument" and works in the polynomial algebra $\mathcal{P}$ of spacetime forms with polynomial coefficients. In the sub-algebra of gauge invariant polynomial forms to which the invariant cochains belong, δ can be split as $\delta = \delta_{free} + \delta_{int}$ where δ_{free} increases by one the number of covariant derivatives of the covariant objects χ and corresponds to $\lambda = 0$, see (2.24). The invariant characteristic cohomology $H(\delta_{free}|d)$ for antighost number > 1 is given by $k\rho^*$. The spectral sequence argument with filtration given by the maximum number of covariant derivatives shows then that the invariant characteristic cohomology is still given by $\alpha_J = k_J\rho^* + \delta\mu_J + d\nu_J$ when λ does not vanish. The corresponding conservation law is of course $\partial_\rho H^{\rho\mu\nu} \approx 0$. $\Box$.

4.2 Elimination of the antifield dependence at ghost number zero and one

We can now show that any solution of (4.1) with ghost number equal to zero or one can be assumed not to depend on the antifields. Since trC^3 is excluded from the γ-cohomology, the ghost number of any $\overline{\omega}_J(C^a)$ ($\neq 1$) is at least equal to 5. In order to construct a solution of (4.1) of ghost number 0 or 1 depending non trivially on the antifields, we need an element of the invariant characteristic cohomology which is at least of antighost 5 or 4. But we have just shown that the non trivial representatives of the invariant characteristic cohomology with highest antighost number are the multiples of ρ^*, whose antighost number is 3. Therefore one cannot find an a_k with $k \geq 1$ which yields a solution of (4.1) depending truly on the antifields. We have thus proved:

Theorem 4.2 *In each class of $H(s|d)$ of ghost 0 or 1, there is an antifield independent representative ($\lambda \neq 0$).*

It is instructive to illustrate this theorem in the case of the deformations of the solution of the master equation obtained by varying the coupling constant

$\lambda \rightarrow \lambda + \delta\lambda$. Since such deformations are consistent, they define elements of $H^0(s|d)$ [19]. In the present case, the deformation reads explicitly

$$\delta\lambda[-\frac{1}{6}H^{\mu\nu\rho}\omega^3_{\mu\nu\rho} + B^{*\mu\nu}\omega^2_{\mu\nu} + \eta^{*\mu}\omega^1_\mu + \frac{1}{3}\rho^* trC^3]. \tag{4.3}$$

For $\lambda = 0$, the antifield dependence is unremovable: the last term cannot be eliminated since trC^3 is non trivial. However, if $\lambda \neq 0$, the antifield dependence should be removable according to the theorem. And indeed, one easily verifies that the above BRST cocycle (modulo d) is in the same class as

$$-\frac{1}{6}\frac{\delta\lambda}{\lambda}H^{\mu\nu\rho}H_{\mu\nu\rho} \tag{4.4}$$

since $\lambda\omega^3_{\mu\nu\rho} = H_{\mu\nu\rho} - \partial_{[\mu}B_{\nu\rho]}$ and $H^{\mu\nu\rho}\partial_{[\mu}B_{\nu\rho]} \approx \partial_\mu(H^{\mu\nu\rho}B_{\nu\rho})$. The representative (4.4) does not involve the antifields.

4.3 Antifield-dependent cohomology

The analysis of the previous section relies crucially upon the assumption that the ghost number is equal to 0 or 1. There exist cocycles (in form degree n) involving non trivially the antifields when the ghost number is ≥ 2. These cocycles fall into two classes. (i) Solutions of type I involve the cocycle ρ^* as term of highest antighost number and are associated to the third order conservation law $\partial_\rho H^{\rho\mu\nu} \approx 0$. (ii) Solutions of type II involve cocycles of antighost number -1 as terms of highest antighost number and are associated to rigid symmetries (ordinary conservation laws).

- Solution of type I

$$\begin{aligned} a &= a_0 + a_1 + a_2 + a_3 && (4.5)\\ &= k_J(\tilde{H}\overline{\omega}'''^J(C^a) + \tilde{B}^*\overline{\omega}''^J(C^a) + \tilde{\eta}^*\overline{\omega}'^J(C^a) + \tilde{\rho}^*\overline{\omega}^J(C^a)). && (4.6)\end{aligned}$$

- Solutions of type II

  Let $a_\Delta = X_{\mu\nu\Delta}B^{*\mu\nu} + Y_{\mu\Delta}A^{*\mu}$ be a complete set of invariant representatives of $H^n_1(\delta|d)$. The a_Δ can be identified with the non-trivial global symmetries [9] of the action (1.1); they satisfy, $\delta a_\Delta + \partial_\mu j^\mu_\Delta = 0$, where

the j^μ_Δ form a complete set of non-trivial conserved currents [9]. The solutions of type II can then be written,

$$\begin{aligned} a &= a_0 + a_1 && (4.7)\\ &= k_J^\Delta((-)^{\epsilon_{\overline{\omega}^J}+1} j^\mu_\Delta \overline{\omega}'^J_\mu(C^a) \\ &\quad +(X_{\mu\nu\Delta}B^{*\mu\nu} + Y_{\mu\Delta}A^{*\mu})\overline{\omega}^J(C^a)). && (4.8)\end{aligned}$$

In the above formulas, the ˜ denotes the form-dual including appropriate multiplicative factors such that,

$$\delta\tilde{B}^* + d\tilde{H} = 0,\ \delta\tilde{\eta}^* + d\tilde{B}^* = 0,\ \delta\tilde{\rho}^* + d\tilde{\eta}^* = 0. \quad (4.9)$$

The $\overline{\omega}'^J, \overline{\omega}''^J, \overline{\omega}'''^J$ are obtained by lifting the $\overline{\omega}^J$ from the bottom of the descent equation,

$$\gamma\overline{\omega}'''^J + d\overline{\omega}''^J = 0,\ \gamma\overline{\omega}''^J + d\overline{\omega}'^J = 0,\ \gamma\overline{\omega}'^J + d\overline{\omega}^J = 0,\ \gamma\overline{\omega}^J = 0. \quad (4.10)$$

The descent exists for all the $\overline{\omega}^J$ because these forms do not depend on trC^3 [20].

5 Antifield independent solutions

5.1 Invariant cohomology of d

In order to examine the antifield independent solutions of (4.1) we need the following result on the invariant cohomology of d:

Theorem 5.3 *Let a be a polynomial in the field strengths and their (covariant) derivatives. Assume the form degree of a to be strictly smaller than n and a to be d-closed: $da = 0$. Then one has $a = \overline{P}(F^a) + db$, where b depends only on the field strengths and their (covariant) derivatives and $\overline{P}$ is an invariant polynomial in the forms F^a which does not contain the quadratic invariant trF^2.*

Proof: When $\lambda = 0$, the invariant cohomology of d is given by the invariant polynomials in the 3-form H and the 2-form F^a. That is, any solution of $da = 0$ which depends only on the field strengths and their derivatives can

be written $a = P(H, F^a) + db(H_{\mu\nu\rho}, \partial_\alpha H_{\mu\nu\rho}, \ldots, F^a_{\mu\nu}, \partial_\alpha F^a_{\mu\nu}, \ldots)$ (See [17] and [18]).

When $\lambda \neq 0$, the invariant cohomology of d is given by the invariant polynomials in the F^a which do not depend on trF^2. Indeed, trF^2 becomes exact in the algebra of *invariant* polynomials, $trF^2 = (\lambda)^{-1} dH$. Furthermore, H disappears also from the cohomology since it is no longer d-closed. $\Box$.

5.2 Results

We can now work out the antifield independent solutions of the Wess-Zumino consistency condition $\gamma a_g + db_{g+1} = 0$. These fall also into two classes. The first one involves the solutions for which the $(n-1)$-form b_{g+1} either vanishes or can be made to vanish by redefinition. The second one involves the solutions that lead to a non trivial descent. The solutions of the first class ($\gamma a_g = 0$) are easily determined since we already know the cohomology of γ. We thus focus on the solutions of the second class, associated to a non trivial descent,

$$\begin{aligned} \gamma a_g + db_{g+1} = 0,\ \gamma b_{g+1} + dc_{g+2} = 0,\ \ldots, \\ \gamma m_s + dn_{s+1} = 0,\ \gamma n_{s+1} = 0. \end{aligned} \tag{5.11}$$

The last term n_{s+1} in the descent is annihilated by γ and thus takes the form $n_{s+1} = \sum_J P_J \overline{\omega}^J(C^a)$ where P_J is an invariant polynomial in the field strength components $F^a_{\lambda\mu}$, $H_{\lambda\mu\nu}$ and their (covariant) derivatives. The next to last equation implies then $dP_J = 0$ and thus, by Theorem **5.3**, the polynomial P_J is actually an invariant polynomial in the forms F^a which may be assumed not to involve trF^2 (H and trF^2 drop out).

This is exactly the form encountered in the pure Yang-Mills case since the variables related to the 2-form $B_{\mu\nu}$ no longer appear. Accordingly, we may proceed along the lines of reference [17] to analyse which cocycles $\sum_J \overline{P}^J(F^a)\overline{\omega}^J(C^a)$ can be lifted all the way up to a solution a_g of the Wess-Zumino consistency condition in degree n. We refer the reader to that work for the details. The only difference is that we start here with a restricted form of the bottom since it involves neither trF^2 nor trC^3. Thus, all the solutions of the second class containing trC^3 or trF^2, or related to them through the descent, become trivial with the introduction of the 2-form $B_{\mu\nu}$ ($\lambda \neq 0$).

5.3 Example in spacetime dimension $d = 10$

We shall illustrate the above procedure in the ten-dimensional case, for ghost number zero and one. For definiteness, we take the gauge group to be $SU(n)$ ($n \geq 6$) so that the primitive forms trC^3, trC^5, trC^7, trC^9 and trC^{11} are all independent. As we have seen, the antifields drop out from the cohomology.

At ghost number one, the only solutions of the Wess-Zumino consistency condition $\gamma a + db = 0$ are of the first type, $\gamma a = 0$ ($b = 0$ by redefinitions). This is because there is no non trivial bottom of the descent with (ghost number + form degree) equal to 10. Thus, the solutions are strictly invariant; they are the invariant polynomials in the individual components $F^a_{\lambda\mu}$, $H_{\lambda\mu\nu}$ and their (covariant) derivatives.

By contrast, there are no strictly invariant solutions at ghost number one and the only solutions of the Wess-Zumino consistency condition are of the second type, associated to a non trivial descent. The possible bottoms must have (ghost number + form degree) equal to 11. The only non trivial ones are trC^{11} and trF^3trC^5. Both can be lifted all the way up to form degree 10 and lead respectively to the irreducible anomaly

$$a_{IRR} = Q^{10,1} \tag{5.12}$$

and the factorizable one,

$$a_F = trF^3Q^{4,1} \tag{5.13}$$

where $Q^{10,1}$ is defined through $dQ^{10,1} + \gamma\omega^{11}_{CS} = 0$, $d\omega^{11}_{CS} = trF^6$ (ω^{11}_{CS} is the eleven-dimensional Chern-Simons form) while $Q^{4,1}$ is the familiar Adler-Bardeen-Jackiw anomaly in four dimensions ($dQ^{4,1} + \gamma\omega^5_{CS} = 0$, $d\omega^5_{CS} = trF^3$). There is no factorizable anomaly related to trF^2 since these become trivial through the coupling to the two-form (Green-Schwarz anomaly cancellation mechanism). Expressions (5.12) and (5.13) are the only solutions of the Wess-Zumino consistency conditions at ghost number one for $SU(n)$ ($n \geq 6$). For other groups, these solutions exist but may be trivial if there is no irreducible three-index or six-index Casimir invariant.

6 Conclusions

In this letter, we have provided the general solution of the Wess-Zumino consistency condition for the Chapline-Manton model, for all ghost numbers

and without use of power counting (which would not help much in any case, since the coupling constants are dimensionful). The antifields have been explicitly included, but have been shown not to bring in new solutions at ghost numbers zero and one.

Our analysis has been carried out in the case of a simple Lie group G and for the quadratic Lagrangian (1.1). Since we have not used power counting, the results can be extended easily to higher-derivative gauge-invariant Lagrangians. They can also be extended to the case where G is the direct product of simple groups by $U(1)$ factors. The simple factors can all be treated as above (if one brings in a 2-form for each such factor). The analysis of the abelian factors is more complicated since they can lead to antifield-dependent solutions even at ghost number zero or one, but it proceeds exactly as in [8].

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